



# **Mark Scheme (Results)**

## **Summer 2026**

Pearson Edexcel International Advanced Level  
In Mechanics M3 (WME03)  
Paper 01A

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\checkmark$  will be used for correct ft
- cao – correct answer only
- cso – correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- $\square$  or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

*(NB specific mark schemes may sometimes override these general principles)*

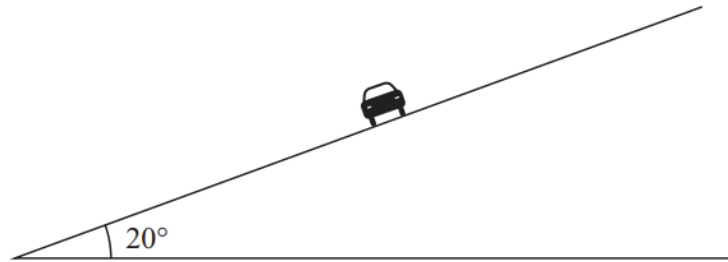
- Rules for M marks:
  - correct no. of terms
  - dimensionally correct
  - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.
  - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

### Mechanics Abbreviations

|      |                                                    |
|------|----------------------------------------------------|
| M(A) | Taking moments about A                             |
| N2L  | Newton's Second Law (Equation of Motion)           |
| NEL  | Newton's Experimental Law (Newton's Law of Impact) |
| HL   | Hooke's Law                                        |
| SHM  | Simple harmonic motion                             |

|      |                                              |
|------|----------------------------------------------|
| PCLM | Principle of conservation of linear momentum |
| RHS  | Right hand side                              |
| LHS  | Left hand side                               |

1.



**Figure 1**

A car moves on a horizontal circular path round a banked bend on a race track.

The radius of the path is 60 m.

The bend is banked at  $20^\circ$  to the horizontal, as shown in Figure 1.

The coefficient of friction between the car and the track is 0.25

The maximum speed at which the car can be driven round the bend without slipping sideways is  $V \text{ m s}^{-1}$

The car and driver are modelled as a single particle of mass 700 kg.

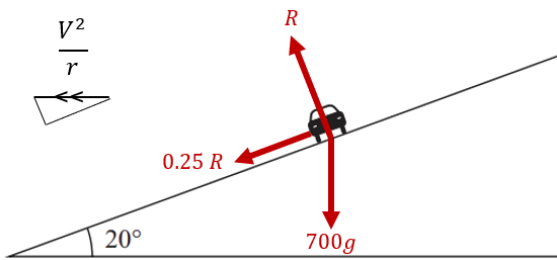
Using the model,

- (a) find the magnitude of the normal reaction between the car and the track when the speed of the car is  $V \text{ m s}^{-1}$

(4)

- (b) find the value of  $V$ .

(4)

| Question Number | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Mark |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 1(a)            |                                                                                                                                                                                                                                                                                                                                                                             |      |
|                 | Form an equation in $R$ only                                                                                                                                                                                                                                                                                                                                                                                                                                  | M1   |
|                 | $R \cos 20^\circ = 700g + 0.25R \sin 20^\circ$                                                                                                                                                                                                                                                                                                                                                                                                                | A1   |
|                 | $(R =) 8030 \text{ or } 8000 \text{ (N)}$                                                                                                                                                                                                                                                                                                                                                                                                                     | A1   |
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                               | (4)  |
| 1(b)            | N2L horizontally                                                                                                                                                                                                                                                                                                                                                                                                                                              | M1   |
|                 | $\frac{700V^2}{60} = R \sin 20^\circ + 0.25R \cos 20^\circ$                                                                                                                                                                                                                                                                                                                                                                                                   | A1   |
|                 | $(V =) 19.9 \text{ or } 20$                                                                                                                                                                                                                                                                                                                                                                                                                                   | A1   |
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                               | (4)  |
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                               | (8)  |
|                 | <b>Notes for Question 1</b>                                                                                                                                                                                                                                                                                                                                                                                                                                   |      |
|                 | See note at end for marking parts (a) and (b) together.<br>Penalise the use of $g = 9.81 \text{ ms}^{-2}$ once per question (first occurrence).<br>Penalise over/under accuracy once per question (first occurrence).                                                                                                                                                                                                                                         |      |
| 1(a)            |                                                                                                                                                                                                                                                                                                                                                                                                                                                               |      |
| M1              | Method to resolve vertically and form an equation in $R$ only using $F = 0.25R$ . All required terms present and no extras.<br>Dimensionally correct. Condone sign errors and sin/cos confusion. Condone $m$ if correctly replaced later.<br><br>Alternative equations, must use $r = 60$ :<br>Perp to slope: $R - 700g \cos 20^\circ = \frac{700V^2}{60} \sin 20^\circ$<br>Parallel to slope: $0.25R + 700g \sin 20^\circ = \frac{700V^2}{60} \cos 20^\circ$ |      |
| A1              | Unsimplified equation in $R$ only with at most one error.                                                                                                                                                                                                                                                                                                                                                                                                     |      |
| A1              | Correct unsimplified equation in $R$ only.                                                                                                                                                                                                                                                                                                                                                                                                                    |      |
| A1              | 3 sf or 2 sf only                                                                                                                                                                                                                                                                                                                                                                                                                                             |      |
| 1(b)            |                                                                                                                                                                                                                                                                                                                                                                                                                                                               |      |
| M1              | Use equation for circular motion to form an equation in $V$ and $R$ only, using $r = 60$ . All required terms present and no extras.<br>Dimensionally correct. Only the relevant terms are resolved but condone sin/cos confusion. Condone sign errors.                                                                                                                                                                                                       |      |
| A1              | Unsimplified equation in $V$ and $R$ with at most one error.                                                                                                                                                                                                                                                                                                                                                                                                  |      |
| A1              | Correct unsimplified equation in $V$ and $R$ only.                                                                                                                                                                                                                                                                                                                                                                                                            |      |
| A1              | 3 sf or 2 sf                                                                                                                                                                                                                                                                                                                                                                                                                                                  |      |
|                 | <b>Note</b> for marking (a) and (b) together.                                                                                                                                                                                                                                                                                                                                                                                                                 |      |

|  |                                                                                                                                                                                                                                                                                                                                                                                                |  |
|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|  | <p>M1A1A1: First relevant equation in <math>V</math> and <math>R</math>. Following rules as above for terms. (A1A0 for an equation with at most one error.)<br/>A1: correct <math>R</math></p> <p>M1A1A1: Second relevant equation in <math>V</math> and <math>R</math>. Following rules as above for terms. (A1A0 for an equation with at most one error.)<br/>A1: correct <math>V</math></p> |  |
|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|

2. **In this question you must show all stages of your working.**  
**Solutions relying entirely on calculator technology are not acceptable.**

A particle is moving along the  $x$ -axis.

At time  $t$  seconds,  $t \geq 0$

- the particle has velocity  $v \text{ m s}^{-1}$  in the positive  $x$  direction
- the particle has acceleration  $\left(3 - \frac{8}{2t+3}\right) \text{ m s}^{-2}$  in the positive  $x$  direction

When  $t = 0$ , the particle passes through the origin with velocity  $2(\ln 3) \text{ m s}^{-1}$  in the positive  $x$  direction.

- (a) Use algebraic integration to show that  $v = pt + q \ln(2t+3) + r \ln 3$ , where  $p$ ,  $q$  and  $r$  are constants to be found.

(4)

- (b) Use algebraic integration to find the distance of the particle from the origin at time  $t = 3$ , giving your answer to 3 significant figures.

(4)

| Question Number             | Answer                                                                                                                                                                                                                          | Mark |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
|                             | Use $v = \int a \, dt$                                                                                                                                                                                                          | M1   |
|                             | $= \lambda t - \mu \ln(2t + 3) \quad (+C)$                                                                                                                                                                                      | A1   |
|                             | $= 3t - 4 \ln(2t + 3) \quad (+C)$                                                                                                                                                                                               | DM1  |
|                             | Use $t = 0, v = 2 \ln 3 \Rightarrow C = \dots$                                                                                                                                                                                  | A1*  |
|                             | $(v =) 3t - 4 \ln(2t + 3) + 6 \ln 3$ *<br>$(p = 3, q = -4, r = 6)$                                                                                                                                                              | (4)  |
| <b>2(b)</b>                 | Use $x = \int v \, dt$ to include integrated terms of the form<br>$\frac{pt^2}{2} + (r \ln 3)t$                                                                                                                                 | M1   |
|                             | Use integration by parts on the term $q \ln(2t + 3)$<br>$\lambda t \ln(2t + 3) + \mu \int \frac{t}{2t + 3} dt$                                                                                                                  | M1   |
|                             | $(x =)$<br>$\frac{3}{2}t^2 + (6 \ln 3)t - 4t \ln(2t + 3) + 4t - 6 \ln(2t + 3) + 6 \ln 3$                                                                                                                                        | A1   |
|                             | 12.3 (m)                                                                                                                                                                                                                        | A1   |
|                             |                                                                                                                                                                                                                                 | (4)  |
|                             |                                                                                                                                                                                                                                 | (8)  |
| <b>Notes for Question 2</b> |                                                                                                                                                                                                                                 |      |
| <b>2(a)</b>                 |                                                                                                                                                                                                                                 |      |
| M1                          | Correct method to obtain $v$ from $a$ by integrating with respect to $t$ .<br>Must reach a form that includes: $\lambda t - \mu \ln(2t + 3)$<br>No need for the constant of integration.<br>M0 for use of $v \frac{dv}{dx}$     |      |
| A1                          | Correct two terms                                                                                                                                                                                                               |      |
| DM1                         | Dependent on previous M. Use initial conditions to evaluate constant of integration.                                                                                                                                            |      |
| A1*                         | Obtain result in given form from correct algebraic integration.<br>The values of $p, q$ and $r$ can be embedded in a correct solution – they do not need to be stated separately.<br>Accept correct terms for $v$ in any order. |      |
| <b>2(b)</b>                 |                                                                                                                                                                                                                                 |      |
| M1                          | Integrate their expression for $v$ with respect to $t$ . Must reach a form that includes: $\frac{pt^2}{2} + (r \ln 3)t$<br>Ignore their integration of $q \ln(2t + 3)$ for this method mark.                                    |      |
| M1                          | Independent. Use integration by parts to integrate $q \ln(2t + 3)$ with respect to $t$ at least as far as: $\lambda t \ln(2t + 3) + \mu \int \frac{t}{2t + 3} dt$                                                               |      |
| A1                          | Any equivalent form including the correct constant of integration (6ln3)                                                                                                                                                        |      |
| A1                          | Must be 3sf.<br>Correct answer must follow correct algebraic integration and the correct value for the constant of integration seen.                                                                                            |      |

|  |                                                                       |  |
|--|-----------------------------------------------------------------------|--|
|  | No need to see substitution of $t = 3$ into their correct expression. |  |
|--|-----------------------------------------------------------------------|--|

3. (a) Use algebraic integration to show that the centre of mass of a uniform

semicircular lamina of radius  $r$  is  $\frac{4r}{3\pi}$  from the centre of the semicircle.

[You may assume, without proof, that the area of a circle of radius  $r$  is  $\pi r^2$ ]

(5)

In the template shown shaded in Figure 2,  $C_1$  is a semicircular lamina with a semicircular lamina  $C_3$  removed. A semicircular lamina  $C_2$  is attached to  $C_1$ .

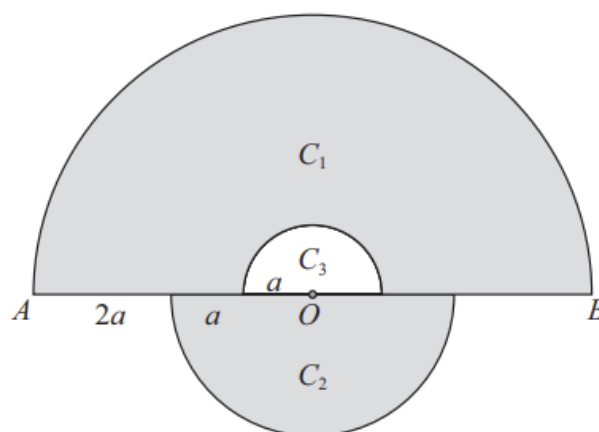


Figure 2

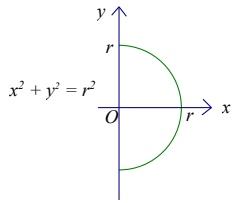
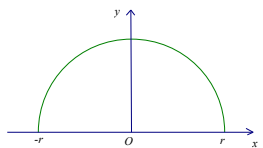
- $C_1$  has radius  $4a$ , diameter  $AB$  and centre  $O$
- $C_2$  has radius  $2a$  and centre  $O$
- $C_3$  has radius  $a$  and centre  $O$

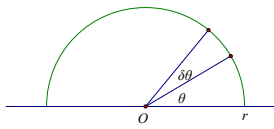
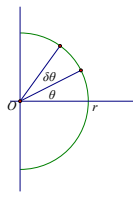
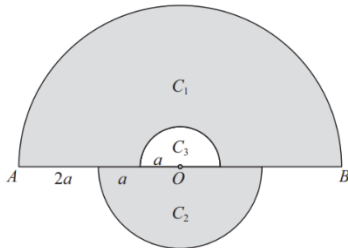
Laminas  $C_1$  and  $C_2$  are made from the same material and lie in the same plane.

The template is freely suspended from  $A$  and hangs in equilibrium with  $AB$  at an angle of  $\theta^\circ$  to the downward vertical.

(b) Find the value of  $\theta$

(7)

| Question Number | Answer                                                                                                                                                                                                                                                                        | Mark |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 3(a)<br>ALT 1   |                                                                                                                                                                                              |      |
|                 | Use $y = \sqrt{r^2 - x^2}$ in an integral of the form $\lambda \int xy \, dx$                                                                                                                                                                                                 | M1   |
|                 | Method to take moments about the y-axis<br>$\lambda \int x(r^2 - x^2)^{\frac{1}{2}} \, dx = \dots (r^2 - x^2)^{\frac{3}{2}}$                                                                                                                                                  | DM1  |
|                 | $= \dots \left(-\frac{1}{3}\right) (r^2 - x^2)^{\frac{3}{2}}$                                                                                                                                                                                                                 | A1   |
|                 | Use limits 0 and $r$ in a complete method<br>$\frac{2 \int_0^r xy \, dx}{\frac{1}{2} \pi r^2}$ <p>For a quadrant:</p> $\frac{\int_0^r xy \, dx}{\frac{1}{4} \pi r^2}$ <p>N.B. Complete solution for a quadrant must include reference to symmetry.</p>                        | DM1  |
|                 | Obtain given answer from correct working<br>$\left( \frac{\frac{2}{3} r^2}{\frac{1}{2} \pi r^2} = \right) \frac{4r}{3\pi} *$                                                                                                                                                  | A1*  |
|                 |                                                                                                                                                                                                                                                                               | (5)  |
| 3(a)<br>ALT2    |                                                                                                                                                                                            |      |
|                 | Use $y = \sqrt{r^2 - x^2}$ in an integral of the form $\lambda \int y^2 \, dx$                                                                                                                                                                                                | M1   |
|                 | Method to take moments about the x-axis<br>$\lambda \int (r^2 - x^2) \, dx = \dots \left( xr^2 - \frac{x^3}{3} \right)$                                                                                                                                                       | DM1  |
|                 | $\frac{1}{2} \left( xr^2 - \frac{x^3}{3} \right)$                                                                                                                                                                                                                             | A1   |
|                 | Use limits $-r$ and $r$ in an expression of the form $ar^2x + bx^3$ in a complete method.<br>$\frac{\int_{-r}^r \frac{1}{2} y^2 \, dx}{\frac{1}{2} \pi r^2} \quad \text{or} \quad \frac{2 \times \int_0^r \frac{1}{2} y^2 \, dx}{\frac{1}{2} \pi r^2}$ <p>For a quadrant:</p> | DM1  |

|                     | $\frac{\int_0^r \frac{1}{2} y^2 \, dx}{\frac{1}{4} \pi r^2}$                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|------|---------------|-------|-------------------------------------|----------------------------|--------|-----------------------------------|----------------------------|-------|----------------------------------|---------------------------|----------|-----------------------------------------|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|----------------|--|--|
|                     | $\left( \frac{\frac{2}{3} r^2}{\frac{1}{2} \pi r^2} = \right) \frac{4r}{3\pi} \quad *$ <p>N.B. Complete solution for a quadrant must include reference to symmetry.</p>                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | A1*  |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | (5)  |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
| 3(a)<br>ALT3        |                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | Use of $\frac{2}{3} r \sin \theta$ or $\frac{2}{3} r \cos \theta$ in the relevant integral                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | M1   |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | Method to take moments about the x-axis or y-axis<br>$\int \frac{1}{2} r^2 \times \frac{2}{3} r \sin \theta \, d\theta$ or $\int \frac{1}{2} r^2 \times \frac{2}{3} r \cos \theta \, d\theta$     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | DM1  |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | $-\frac{1}{3} r^3 \cos \theta$ or $\frac{1}{3} r^3 \sin \theta$                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | A1   |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | Use limits 0 and $\pi$ in an expression of the form $\mu r^3 \cos \theta$ or limits $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ in an expression of the form $\mu r^3 \sin \theta$ in a complete method. |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | DM1  |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | $\left( \frac{\frac{2}{3} r^2}{\frac{1}{2} \pi r^2} = \right) \frac{4r}{3\pi} \quad *$                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | A1*  |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | (5)  |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
| 3(b)                |                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   | <table><thead><tr><th></th><th>area</th><th>C of M from O</th></tr></thead><tbody><tr><td>large</td><td><math>\frac{1}{2} \pi \times 16a^2</math> [16]</td><td><math>\frac{4 \times 4a}{3\pi}</math></td></tr><tr><td>medium</td><td><math>\frac{1}{2} \pi \times 4a^2</math> [4]</td><td><math>\frac{4 \times 2a}{3\pi}</math></td></tr><tr><td>small</td><td><math>\frac{1}{2} \pi \times a^2</math> [1]</td><td><math>\frac{4 \times a}{3\pi}</math></td></tr><tr><td>template</td><td><math>\frac{1}{2} \pi \times 19a^2</math> [16+4-1]</td><td><math>d</math></td></tr></tbody></table> |                                                                                   | area | C of M from O | large | $\frac{1}{2} \pi \times 16a^2$ [16] | $\frac{4 \times 4a}{3\pi}$ | medium | $\frac{1}{2} \pi \times 4a^2$ [4] | $\frac{4 \times 2a}{3\pi}$ | small | $\frac{1}{2} \pi \times a^2$ [1] | $\frac{4 \times a}{3\pi}$ | template | $\frac{1}{2} \pi \times 19a^2$ [16+4-1] | $d$ | <table><tbody><tr><td rowspan="4">B1<br/>mass<br/>ratio</td></tr><tr><td rowspan="3">B1<br/>distance</td></tr><tr><td></td></tr><tr><td></td></tr></tbody></table> | B1<br>mass<br>ratio | B1<br>distance |  |  |
|                     |                                                                                                                                                                                                   | area                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | C of M from O                                                                     |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | large                                                                                                                                                                                             | $\frac{1}{2} \pi \times 16a^2$ [16]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $\frac{4 \times 4a}{3\pi}$                                                        |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | medium                                                                                                                                                                                            | $\frac{1}{2} \pi \times 4a^2$ [4]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | $\frac{4 \times 2a}{3\pi}$                                                        |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | small                                                                                                                                                                                             | $\frac{1}{2} \pi \times a^2$ [1]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{4 \times a}{3\pi}$                                                         |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
| template            | $\frac{1}{2} \pi \times 19a^2$ [16+4-1]                                                                                                                                                           | $d$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
| B1<br>mass<br>ratio |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     | B1<br>distance                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |
|                     |                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   |      |               |       |                                     |                            |        |                                   |                            |       |                                  |                           |          |                                         |     |                                                                                                                                                                    |                     |                |  |  |

|                            |                                                                                                                                                                                                                                               |      |
|----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
|                            | Moments about $AB$                                                                                                                                                                                                                            | M1   |
|                            | $\frac{1}{2}\pi \times 19a^2d =$ $\frac{1}{2}\pi \times 16a^2 \times \frac{16a}{3\pi} - \frac{1}{2}\pi \times 4a^2 \times \frac{8a}{3\pi} - \frac{1}{2}\pi \times a^2 \times \frac{4a}{3\pi}$                                                 | A1   |
|                            | $\left(19d = \frac{256a}{3\pi} - \frac{32a}{3\pi} - \frac{4a}{3\pi}\right) \quad d = \frac{220a}{57\pi}$                                                                                                                                      | A1   |
|                            | $\tan \theta^\circ = \frac{\left(\frac{220a}{57\pi}\right)}{4a}$                                                                                                                                                                              | M1   |
|                            | $\theta = 17$                                                                                                                                                                                                                                 | A1   |
|                            |                                                                                                                                                                                                                                               | (7)  |
|                            |                                                                                                                                                                                                                                               | (12) |
|                            | <b>Notes for Question 3</b>                                                                                                                                                                                                                   |      |
| <b>3(a)</b><br><b>ALT1</b> |                                                                                                                                                                                                                                               |      |
| M1                         | Use $y = \sqrt{r^2 - x^2}$ in an integral of the form $\lambda \int xy \, dx$ .<br>No need to attempt integration.                                                                                                                            |      |
| DM1                        | Dependent on the previous M. Method to take moments about the $y$ -axis. Ignore constants. Integrate to the required form.<br>$\lambda \int x(r^2 - x^2)^{\frac{1}{2}} dx = \dots (r^2 - x^2)^{\frac{3}{2}}$                                  |      |
| A1                         | Correct integration (constant may be processed later)                                                                                                                                                                                         |      |
| DM1                        | Dependent on the two preceding M marks<br>Use limits 0 and $r$ in a complete method $\frac{2 \int_0^r xy \, dx}{\frac{1}{2}\pi r^2}$<br>e.g. $\frac{0 + \frac{2}{3}r^3}{\frac{1}{2}\pi r^2}$<br>Quadrant method must use $\frac{1}{4}\pi r^2$ |      |
| A1*                        | Obtain given answer from complete and correct algebraic integration.<br>N.B. Complete solution for a quadrant must include reference to symmetry.                                                                                             |      |
| <b>3(a)</b><br><b>ALT2</b> |                                                                                                                                                                                                                                               |      |
| M1                         | Integrate $y = \sqrt{r^2 - x^2}$ in an integral of the form $\lambda \int y^2 \, dx$ . No need to attempt integration                                                                                                                         |      |
| DM1                        | Dependent on the first M1. Method to take moments about the $x$ -axis. Ignore constants. Integrate to the required form.                                                                                                                      |      |

|                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |
|----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|                            | $\lambda \int r^2 - x^2 \, dx = \dots x r^2 - \frac{x^3}{3}$                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |
| A1                         | Correct integration (constant may be processed later)                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |
| DM1                        | <p>Dependent on the two preceding M marks.</p> <p>Use limits <math>-r</math> and <math>r</math> in a complete method</p> $\frac{\int_{-r}^r \frac{1}{2} y^2 \, dx}{\frac{1}{2} \pi r^2} \quad \text{or} \quad \frac{2 \times \int_0^r \frac{1}{2} y^2 \, dx}{\frac{1}{2} \pi r^2}$ <p>e.g. <math>\frac{\frac{1}{2} \left( r^3 - \frac{1}{3} r^3 - \left( -r^3 + \frac{1}{3} r^3 \right) \right)}{\frac{1}{2} \pi r^2}</math></p> <p>Quadrant method must use <math>\frac{1}{4} \pi r^2</math></p> |  |
| A1*                        | <p>Obtain given answer from complete and correct algebraic integration.</p> <p>N.B. Complete solution for a quadrant must include reference to symmetry.</p>                                                                                                                                                                                                                                                                                                                                      |  |
| <b>3(a)</b><br><b>ALT3</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |
| M1                         | Use of $\frac{2}{3} r \sin \theta$ or $\frac{2}{3} r \cos \theta$ in the relevant integral. No need to attempt integration.                                                                                                                                                                                                                                                                                                                                                                       |  |
| DM1                        | <p>Dependent on the first M1. Ignore constants. Take moments and integrate to the required form.</p> $\dots r^3 \cos \theta \quad \text{or} \quad \dots r^3 \sin \theta$                                                                                                                                                                                                                                                                                                                          |  |
| A1                         | $-\frac{1}{3} r^3 \cos \theta$ or $\frac{1}{3} r^3 \sin \theta$ (constant may be processed later)                                                                                                                                                                                                                                                                                                                                                                                                 |  |
| DM1                        | <p>Dependent on the two preceding M marks.</p> <p>Use limits 0 and <math>\pi</math> in an expression of the form <math>\gamma r^3 \cos \theta</math> or use limits <math>-\frac{1}{2} \pi</math> and <math>\frac{1}{2} \pi</math> in an expression of the form <math>\gamma r^3 \sin \theta</math> in a complete method.</p>                                                                                                                                                                      |  |
| A1*                        | Obtain given answer from complete and correct algebraic integration.                                                                                                                                                                                                                                                                                                                                                                                                                              |  |
| <b>(b)</b>                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |
| B1                         | Correct mass ratios                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |
| B1                         | Correct distances (ignore signs)                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |
| M1                         | Use their mass ratios and distances to take moments about $AB$ or a parallel axis. Must form a dimensionally <b>consistent</b> equation. Condone sign errors. Accept mass ratio of 21 instead of 19.                                                                                                                                                                                                                                                                                              |  |
| A1                         | Correct unsimplified equation.                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |  |
| A1                         | Accept, 1.228... $a$                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |  |
| M1                         | Use their $d$ with trig to form a dimensionally correct equation in a relevant angle. Accept the reciprocal and consistent cancelled $a$ 's.                                                                                                                                                                                                                                                                                                                                                      |  |

|    |                                                                                       |  |
|----|---------------------------------------------------------------------------------------|--|
|    | $\tan \theta^\circ = \frac{\frac{220a}{57\pi}}{4a} \left( = \frac{55}{57\pi} \right)$ |  |
| A1 | Or better: 17.073... (only)                                                           |  |

4. A light elastic **spring**, of natural length  $4a$  and modulus of elasticity  $3mg$ , has one end attached to a fixed point  $A$ .  
A particle of mass  $m$  is attached to the other end of the spring. The particle hangs vertically below  $A$ .

The particle is pulled vertically downwards to the point  $B$ , where  $AB = 7a$ , and released from rest.

Air resistance is ignored.

- (a) Find the acceleration of the particle at the instant it is released from rest. (3)

- (b) By considering energy, find in terms of  $g$  and  $a$ , the speed of the particle when it is a distance  $5a$  below  $A$ . (5)

The particle first comes to instantaneous rest at the point  $C$ , where  $AC = ka$  and  $k$  is a constant.

- (c) By considering energy, find the value of  $k$ . (5)

| Question Number | Answer                                                                                                                                                                                                            | Mark |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 4(a)            |                                                                                                                                                                                                                   |      |
|                 | Equation of motion at $B$ ( $x = 3a$ )                                                                                                                                                                            | M1   |
|                 | $\frac{3mg(3a)}{4a} - mg = \pm m\ddot{x}$                                                                                                                                                                         | A1   |
|                 | $(acc =) \pm \frac{5g}{4}$                                                                                                                                                                                        | A1   |
|                 |                                                                                                                                                                                                                   | (3)  |
| 4(b)            | Either $\frac{3mg(3a)^2}{8a}$ or $\frac{3mg(a)^2}{8a}$ seen                                                                                                                                                       | B1   |
|                 | Conservation of energy to the point where $x = a$                                                                                                                                                                 | M1   |
|                 | $\frac{3mg(3a)^2}{8a} = mg(2a) + \frac{3mg(a)^2}{8a} + \frac{1}{2}mv^2$                                                                                                                                           | A1   |
|                 | $v = \sqrt{2ga}$                                                                                                                                                                                                  | A1   |
|                 |                                                                                                                                                                                                                   | (5)  |
| 4(c)            | A second EPE term ( <b>different</b> to the one awarded a mark in part (b))                                                                                                                                       | B1   |
|                 | Conservation of energy to $C$ ( $v = 0$ )                                                                                                                                                                         | M1   |
|                 | Correct energy equation to $C$<br>e.g. From $B$ to $C$                                                                                                                                                            | A1   |
|                 | <ul style="list-style-type: none"> <li>if <math>x</math> is the compression in the spring at <math>C</math><br/> <math display="block">\frac{3mg(3a)^2}{8a} = mg(x + 3a) + \frac{3mg(x)^2}{8a}</math> </li> </ul> |      |
|                 | <ul style="list-style-type: none"> <li>if <math>x</math> is the extension in the spring at <math>C</math><br/> <math display="block">\frac{3mg(3a)^2}{8a} = mg(3a - x) + \frac{3mg(x)^2}{8a}</math> </li> </ul>   |      |
|                 | <ul style="list-style-type: none"> <li>if <math>x = \pm(4a - ka)</math> is the extension<br/> <math display="block">\frac{3mg(3a)^2}{8a} = mg(7a - ka) + \frac{3mg(ka - 4a)^2}{8a}</math> </li> </ul>             |      |
|                 | Solve to obtain $k =$                                                                                                                                                                                             | DM1  |
|                 | $k = \frac{11}{3} \text{ (only)}$                                                                                                                                                                                 | A1   |
|                 |                                                                                                                                                                                                                   | (5)  |
|                 |                                                                                                                                                                                                                   | (13) |
|                 | <b>Notes for Question 4</b>                                                                                                                                                                                       |      |
| 4(a)            |                                                                                                                                                                                                                   |      |

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <b>M1</b>   | Use Hooke's Law in an equation of motion at $B$ . Tension must have an extension of $3a$ . All required terms present and no extras. Dimensionally correct. Condone sign errors. Condone $a$ for acceleration but M0 if it is treated as a length.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |  |
| <b>A1</b>   | Correct unsimplified equation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |
| <b>A1</b>   | Accept 12, 12.3 ( $\text{ms}^{-2}$ )                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |  |
| <b>4(b)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |
| <b>B1</b>   | One correct EPE term                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |  |
| <b>M1</b>   | Form an equation using conservation of energy from $B$ to $x = a$ . All required terms present and no extras. Dimensionally correct. Condone sign errors.<br>EPE must have the form $\frac{\lambda x^2}{\mu a}$ where $x = a$ and $\mu$ is a constant (condone $\mu = 1$ ).<br>GPE must have the form $mgh$ where $h = 2a$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |  |
| <b>A1</b>   | Unsimplified equation with at most one error                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |
| <b>A1</b>   | Correct unsimplified equation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |
| <b>A1</b>   | Or equivalent in terms of $a$ and $g$<br>Accept a correctly rounded decimal instead of $\sqrt{2}$ but must have $\sqrt{ga}$ . Calculator display for $\sqrt{2} = 1.414213562$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |
| <b>4(c)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |
| <b>B1</b>   | A correct relevant EPE term, it must be <b>different</b> to the one awarded in (b) and it must be used in (c).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |  |
| <b>M1</b>   | Form a dimensionally correct conservation of energy equation<br>Condone sign errors.<br>EPE term must have the form $\frac{\lambda x^2}{\mu a}$ where $x$ is their extension and $\mu$ is a constant (condone $\mu = 1$ ).<br>GPE term must have the form $mgh$ where $h$ is a height in terms of $a$ and/or $x$<br><br>All required terms present and no extras.<br><ul style="list-style-type: none"> <li><b>B to C:</b> 2 EPE terms and GPE</li> <li><b>Alternative point to C:</b> KE, GPE, 2 EPE (but 1EPE if using natural length position)</li> </ul> <b>Equilibrium to C:</b> $x = \frac{4a}{3}$ , $v^2 = \frac{25}{12}ga$<br>leads to $\frac{3mg(\frac{4a}{3})^2}{8a} + \frac{1}{2}m\left(\frac{25}{12}ag\right) = mg\left(\frac{16a}{3} - ka\right) + \frac{3mg(ka-4a)^2}{8a}$<br><br><b>Natural length to C:</b> $x = 0$ , $v^2 = \frac{3}{4}ga$<br>leads to $\frac{1}{2}m\left(\frac{3}{4}ag\right) = mg(4a - ka) + \frac{3mg(ka-4a)^2}{8a}$<br><br><b>Point in (b) to C:</b> $x = a$ , $v^2 = 2ga$<br>leads to $\frac{3mg(a)^2}{8a} + \frac{1}{2}m(2ga) = mg(5a - ka) + \frac{3mg(ka-4a)^2}{8a}$ |  |
| <b>A1</b>   | Correct unsimplified equation (using their letter for length)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |
| <b>DM1</b>  | Dependent on first M1. Solve to find a value for $k$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |  |
| <b>A1</b>   | 3.7 or better                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |

5.

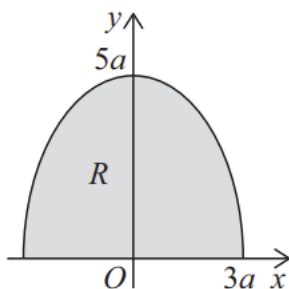


Figure 3

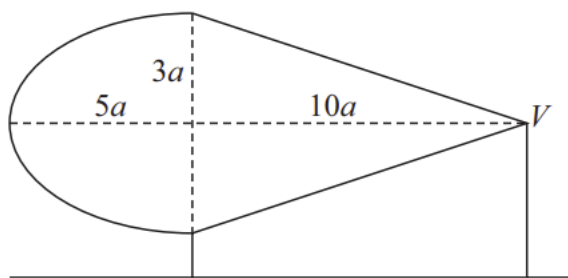


Figure 4

The region  $R$ , shown shaded in Figure 3, is the region enclosed between the  $x$ -axis and

the curve with equation  $\frac{x^2}{9a^2} + \frac{y^2}{25a^2} = 1$

Region  $R$  is rotated through  $\pi$  radians about the  $y$ -axis to form a solid of revolution. A uniform solid  $S$  has the same shape and size as this solid of revolution.

The volume of  $S$  is  $30\pi a^3$

The distance of the centre of mass of  $S$  from  $O$  is  $d$ .

(a) Use algebraic integration to show that  $d = \frac{15}{8}a$ .

(6)

The solid  $S$  is joined to a uniform solid right circular cone  $C$  that is made of the **same material** as  $S$  to form a model.

The cone has vertex  $V$ , height  $10a$  and base radius  $3a$ .

The plane face of  $S$  and the plane face of  $C$  coincide.

The model is supported on two light vertical rods.

- the first rod has one end attached to the model at  $V$  and the other end attached to horizontal ground
- the second rod has one end attached to the model at a point on the rim of the common plane face and the other end attached to the horizontal ground

The model rests in equilibrium with its axis of symmetry horizontal and in the same vertical plane as the two rods, as shown in Figure 4.

Given that the weight of the model is  $2W$

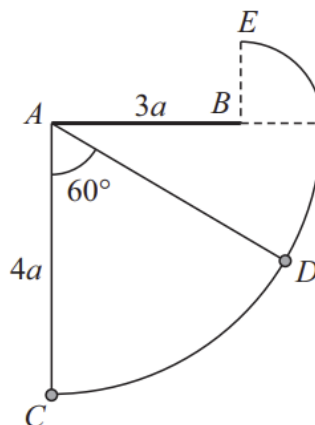
(b) find, in terms of  $W$ , the thrust in the rod attached to the model at  $V$ .

(4)

| Question Number      | Answer                                                                                                                                                                                                                | Mark |
|----------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 5(a)                 | Take moments about the $y$ -axis, $(\pi) \int y x^2 dy$ with $x^2$ replaced in terms of $y$ .<br><b>M0</b> for moments about the $x$ -axis.                                                                           | M1   |
|                      | Integrate with respect to $y$ to the form<br>$(\pi)a^2(p y^2 + q y^4)$                                                                                                                                                | M1   |
|                      | $= (\pi)a^2 \left( \frac{9y^2}{2} - \frac{9y^4}{100a^2} \right)$                                                                                                                                                      | A1   |
|                      | Use limits 0 and $5a$<br>$\pi \left( \frac{9a^2 \times 25a^2}{2} - \frac{9a^2 \times 625a^4}{100a^2} \right)$                                                                                                         | DM1  |
|                      | Complete the process to find the centre of mass                                                                                                                                                                       | DM1  |
|                      | $\left( d = \frac{9a}{30} \times \frac{25}{4} \right) = \frac{15}{8} a$ *                                                                                                                                             | A1*  |
|                      |                                                                                                                                                                                                                       | (6)  |
| 5(b)                 | A complete method to obtain an equation in $W$ and $T_v$ only using the cone and dome.                                                                                                                                | M1   |
|                      | For example,<br>• cone and dome separately: $W \frac{5a}{2} = W \frac{15a}{8} + 10aT_v$                                                                                                                               | A1   |
|                      | • cone and dome combined: $2W \times \frac{5a}{16} = 10aT_v$                                                                                                                                                          | A1   |
|                      | $(T_v =) \frac{5W}{80} = \frac{W}{16}$                                                                                                                                                                                | A1   |
|                      |                                                                                                                                                                                                                       | (4)  |
|                      |                                                                                                                                                                                                                       | (10) |
| Notes for Question 5 |                                                                                                                                                                                                                       |      |
| 5(a)                 |                                                                                                                                                                                                                       |      |
| M1                   | Take moments about the $y$ axis, using $(\pi) \int y x^2 dy$ . Substitute their equation for $x^2$ into the correct formula.<br><b>M0 for moments taken about the <math>x</math>-axis so no further marks in (a).</b> |      |
| M1                   | Integration with respect to $y$ to reach the form $(py^2 + qy^4)$ .<br>Ignore constants.<br>$\pi \int 9a^2 y \left( 1 - \frac{y^2}{25a^2} \right) dy \left( = 9a^2 \pi \int y - \frac{y^3}{25a^2} dy \right)$         |      |
| A1                   | Correct integration.                                                                                                                                                                                                  |      |
| DM1                  | Dependent on the preceding M. Use the limits 0 and $5a$ in their integrated expression. Ignore constants. Allow if substitution of 0 is not seen.                                                                     |      |
| DM1                  | Dependent on both preceding M's. Divide by the given volume in a complete method. Condone consistent missing $\pi$ and consistent $\rho$ (both numerator and denominator or neither).                                 |      |
| A1*                  | Obtain given answer from correct working                                                                                                                                                                              |      |
| 5(b)                 |                                                                                                                                                                                                                       |      |

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |  |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <b>M1</b> | <p>Correct method involves an attempt to find the centre of mass of the cone or the combined solid in terms of <math>a</math> <b>and</b> a moments equation in <math>W</math> and <math>T_v</math>.</p> <p>The moments equation must be dimensionally correct with all required terms present and no extras. Accept consistent missing <math>a</math>'s in the moments equation. Condone sign errors.</p> <p>If more than one equation is formed, <math>T_p</math> must be eliminated to earn the method mark.</p> <p>Alternative equations:</p> <p>Vert: <math>T_p + T_v = 2W</math></p> <p>M(<math>V</math>) combined <math>10aT_p = \left(10a - \frac{5}{16}a\right) \times 2W</math></p> <p>M(<math>V</math>) separate <math>10aT_p = \frac{30a}{4} \times W + \left(10a + \frac{15a}{8}\right)W</math></p> <p>M (plane face): <math>2W \times \frac{5a}{16} = 10aT_v</math></p> <p><b>Note</b> the cone and the dome both have weight <math>W</math>.</p> <p>C of M for the model is <math>\frac{5a}{16}</math> from the common plane face (towards the vertex of the cone).</p> |  |
| <b>A1</b> | Unsimplified equation in $W$ and $T_v$ with at most one error                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |  |
| <b>A1</b> | Correct unsimplified equation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |  |
| <b>A1</b> | Correct answer, accept $0.0625W$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |

6.



**Figure 5**

A thin fixed horizontal shelf  $AB$  has length  $3a$ .

A particle of mass  $m$  is attached to one end of a light inextensible string of length  $4a$ .

The other end of the string is attached to the underneath of the shelf at  $A$ .

The particle is hanging at rest at  $C$ , vertically below  $A$ , when it is projected horizontally with speed  $u$ .

The particle starts to move, with the string taut, in a vertical plane that is perpendicular to the plane of the shelf. The edge of the shelf is perpendicular to the circular path of the particle.

Air resistance is ignored.

When the string is horizontal, the string wraps round the end of the shelf and the particle continues to move in the vertical plane.

The particle follows the path  $CDE$ , as shown in Figure 5.

The point  $D$  on the circular path of the particle is such that angle  $CAD = 60^\circ$

- (a) Find, in terms of  $m$ ,  $u$ ,  $a$  and  $g$ , the tension in the string at the instant when the particle is at  $D$ .

(5)

The point  $E$  is vertically above  $B$ , with  $BE = a$ .

Given that the particle is still moving in a vertical circle when it reaches  $E$

- (b) show that  $u \geq \sqrt{11ga}$

(3)

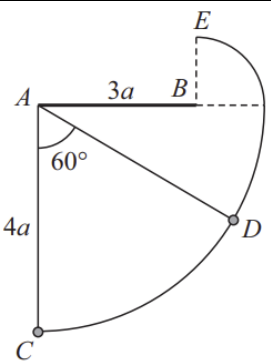
At the instant when the particle is at  $E$ , the particle breaks free from the string.

The particle moves freely under gravity and first lands on the shelf at the point  $F$ .

Given that  $u = \sqrt{12ga}$

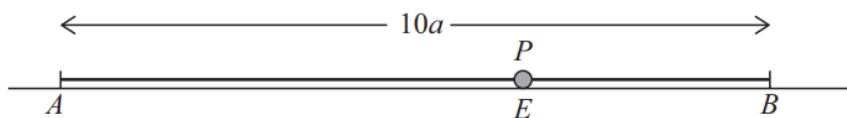
- (c) find, in terms of  $a$ , the distance  $BF$ .

(4)

| Question Number | Answer                                                                                                                                                      | Mark |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
|                 |                                                                            |      |
| 6(a)            | Equation for circular motion                                                                                                                                | M1   |
|                 | $T - mg \cos 60^\circ = \frac{mv^2}{4a}$                                                                                                                    | A1   |
|                 | Conservation of energy from C to D                                                                                                                          | M1   |
|                 | $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg \times (4a - 4a \cos 60^\circ)$                                                                                     | A1   |
|                 | $T = \frac{mu^2}{4a} - \frac{mg}{2}$                                                                                                                        | A1   |
|                 |                                                                                                                                                             | (5)  |
| 6(b)            | Conservation of energy from C to E<br>$\frac{1}{2}mu^2 = \frac{1}{2}mw^2 + mg \times 5a$<br><b>and</b> N2L for circular motion<br>$T = \frac{mw^2}{a} - mg$ | M1   |
|                 | Use of condition for circular motion to form an inequality in $a$ , $g$ and $u$ or $w$ only<br>$(T \geq 0 \dots) \quad w^2 \geq ag$                         | M1   |
|                 | Complete solution must include sight of $T \geq 0$ , correct inequality signs and correct working that leads to<br>$u \geq \sqrt{11ga} \quad *$             | A1*  |
|                 |                                                                                                                                                             | (3)  |
|                 |                                                                                                                                                             |      |
| 6(c)            | Use $u^2 = 12ga$ to obtain $w^2 = 2ag$ <b>or</b> $w = \sqrt{2ga}$                                                                                           | B1ft |
|                 | Vertical motion to find the time taken<br>$a = \frac{1}{2}gt^2, \quad t = \dots \left( \sqrt{\frac{2a}{g}} \right)$                                         | M1   |
|                 | Horizontal motion to find the distance $s = \sqrt{2ga} \times t$                                                                                            | M1   |
|                 | $= 2a$                                                                                                                                                      | A1   |
|                 |                                                                                                                                                             | (4)  |
|                 |                                                                                                                                                             | (13) |
|                 | <b>Notes for Question 6</b>                                                                                                                                 |      |
| 6(a)            |                                                                                                                                                             |      |
| M1              | N2L with circular form of acceleration and $r = 4a$ . Accept $4a$ replaced later in working. Dimensionally correct. All required                            |      |

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |  |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|             | terms present and no extra. Condone sign errors and sin/cos confusion.                                                                                                                                                                                                                                                                                                                                                                                                     |  |
| A1          | Correct unsimplified equation                                                                                                                                                                                                                                                                                                                                                                                                                                              |  |
| M1          | Conservation of energy from $B$ to $C$ . Dimensionally correct. All required terms present and no extra. Condone sign errors and sin/cos confusion. All energy terms with the correct structure. Accept vertical height as $2a$ without seeing $(4a - 4a \cos 60^\circ)$ .                                                                                                                                                                                                 |  |
| A1          | Correct unsimplified equation                                                                                                                                                                                                                                                                                                                                                                                                                                              |  |
| A1          | Accept equivalent but must have $mg$ terms collected.                                                                                                                                                                                                                                                                                                                                                                                                                      |  |
| <b>6(b)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |  |
| M1          | Equations for conservation of energy and circular motion at $E$ . Dimensionally correct. All required terms present and no extra. Condone sign errors.<br><br>Accept two energy equations where the journey from $C$ to $E$ is broken down into $C$ to level with $B$ then level with $B$ to $E$ .<br><br>$C$ to $B$ : $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(4a) \Rightarrow v^2 = u^2 - 8ag$<br>and<br>$B$ to $E$ : $\frac{1}{2}m(u^2 - 8ag) = \frac{1}{2}mw^2 + mg(a)$ |  |
| M1          | Use correct condition for circular motion to form an inequality in $a$ , $g$ and $w$ or $u$ . For the method mark only, condone $T \geq 0$ not stated and condone strict inequality sign.                                                                                                                                                                                                                                                                                  |  |
| A1*         | Obtain given answer from complete and correct working. This must include sight of $T \geq 0$ and the correct inequality signs throughout.                                                                                                                                                                                                                                                                                                                                  |  |
| <b>6(c)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |  |
| B1ft        | Use $u^2 = 12ga$ to obtain $w^2 = 2ga$ <b>or</b> $w = \sqrt{2ga}$<br>Follow through is for their $w$ following substitution of $u^2 = 12ga$ into their expression for $v^2$ in their energy equation in (b).                                                                                                                                                                                                                                                               |  |
| M1          | Vertical motion: correct use of <i>suvat</i> to find an expression for $t$ in terms of $a$ and $g$                                                                                                                                                                                                                                                                                                                                                                         |  |
| M1          | Horizontal motion to find the distance $BF$                                                                                                                                                                                                                                                                                                                                                                                                                                |  |
| A1          | Correct only                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |

7.



**Figure 6**

The fixed points  $A$  and  $B$  lie on a smooth horizontal surface with  $AB = 10a$ .

A particle  $P$  has mass  $m$ .

One end of a light elastic string, of natural length  $5a$  and modulus of elasticity  $\frac{1}{3}mg$ ,

is attached to  $P$  and the other end is attached to  $A$ .

One end of another light elastic string, of natural length  $3a$  and modulus of elasticity

$\frac{2}{5}mg$ , is attached to  $P$  and the other end is attached to  $B$ .

The particle  $P$  is resting in equilibrium at the point  $E$  on the surface, as shown in Figure 6.

- (a) Show that  $EB = \frac{11a}{3}$  (3)

Particle  $P$  is now held on the surface at the point  $C$ , between  $A$  and  $B$ , where  $CB = 3a$ , and released from rest.

- (b) Show that  $P$  moves with simple harmonic motion about  $E$ . (4)

- (c) Show that the simple harmonic motion has period  $2\pi\sqrt{\frac{5a}{g}}$  (2)

In any one complete oscillation of the simple harmonic motion, the particle is less than

a distance  $\frac{1}{3}a$  from  $E$  for a time  $S$ .

- (d) Find  $S$  in terms of  $\pi$ ,  $a$  and  $g$ . (3)

| Question Number | Answer                                                                                                                                                                                                                                                                                                                                                                                     | Mark |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 7(a)            | <p> <i>nat. length</i> <math>5a</math>                      <i>nat. length</i> <math>3a</math><br/> <i>modulus</i> <math>\frac{1}{3}mg</math>                      <i>modulus</i> <math>\frac{2}{5}mg</math> </p>                                                                                                                                                                          |      |
|                 | Equate the tensions                                                                                                                                                                                                                                                                                                                                                                        | M1   |
|                 | Correct equation with unknown extensions that sum to $2a$<br>e.g. $\frac{\frac{2}{5}mge}{3a} = \frac{\frac{1}{3}mg(7a - e - 5a)}{5a}$                                                                                                                                                                                                                                                      | A1   |
|                 | Solve their equation correctly with working that leads<br>$(EB =) \frac{11a}{3} *$                                                                                                                                                                                                                                                                                                         | A1*  |
|                 |                                                                                                                                                                                                                                                                                                                                                                                            | (3)  |
| 7(b)            | N2L for general position using difference of tensions<br>$\pm(T_A - T_B) = -m\ddot{x}$                                                                                                                                                                                                                                                                                                     | M1   |
|                 | Correct equation<br>e.g. $\bullet \frac{\frac{mg}{3}\left(\frac{19a}{3} + x - 5a\right)}{5a} - \frac{\frac{2mg}{5}\left(\frac{11a}{3} - x - 3a\right)}{3a} = -m\ddot{x}$                                                                                                                                                                                                                   | A1   |
|                 | $\bullet \frac{\frac{mg}{3}\left(\frac{4a}{3} + x\right)}{5a} - \frac{\frac{2mg}{5}\left(\frac{2a}{3} - x\right)}{3a} = -m\ddot{x}$                                                                                                                                                                                                                                                        | A1   |
|                 | Reach given conclusion from complete and correct working.<br>This must include all of: <ul style="list-style-type: none"> <li>at least one step between the initial N2L equation in <math>x</math> and the conclusion</li> <li>reaching the correct form <math>\ddot{x} = -\frac{g}{5a}x</math></li> <li>concluding SHM e.g. <math>\ddot{x} = -\frac{g}{5a}x</math> Hence SHM *</li> </ul> | A1*  |
|                 |                                                                                                                                                                                                                                                                                                                                                                                            | (4)  |
| 7(c)            | Use of periodic time $= \frac{2\pi}{\omega}$                                                                                                                                                                                                                                                                                                                                               | M1   |
|                 | $= 2\pi \sqrt{\frac{5a}{g}} *$                                                                                                                                                                                                                                                                                                                                                             | A1*  |
|                 |                                                                                                                                                                                                                                                                                                                                                                                            | (2)  |
| 7(d)            | Correct method<br>$x = \frac{2}{3}a \sin \sqrt{\frac{g}{5a}}t = \frac{1}{3}a$                                                                                                                                                                                                                                                                                                              | M1   |

|                             |                                                                                                                                                                                                                                                                                                                                   |      |
|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
|                             | $t = \frac{\pi}{6} \sqrt{\frac{5a}{g}}$                                                                                                                                                                                                                                                                                           | A1   |
|                             | $(S =) 4t = \frac{2\pi}{3} \sqrt{\frac{5a}{g}}$                                                                                                                                                                                                                                                                                   | A1   |
|                             |                                                                                                                                                                                                                                                                                                                                   | (3)  |
| <b>7(d)</b><br><b>ALT</b>   | Correct method                                                                                                                                                                                                                                                                                                                    | M1   |
|                             | $x = \frac{2}{3}a \cos \sqrt{\frac{g}{5a}}t = \frac{1}{3}a$                                                                                                                                                                                                                                                                       |      |
|                             | $t = \frac{\pi}{3} \sqrt{\frac{5a}{g}}$                                                                                                                                                                                                                                                                                           | A1   |
|                             | $(S =) 2\pi \sqrt{\frac{5a}{g}} - 4t = \frac{2\pi}{3} \sqrt{\frac{5a}{g}}$                                                                                                                                                                                                                                                        | A1   |
|                             |                                                                                                                                                                                                                                                                                                                                   | (3)  |
|                             |                                                                                                                                                                                                                                                                                                                                   | (12) |
| <b>Notes for Question 7</b> |                                                                                                                                                                                                                                                                                                                                   |      |
| <b>(a)</b>                  |                                                                                                                                                                                                                                                                                                                                   |      |
| M1                          | Equate the tensions in each string. Dimensionally correct.<br>Tensions must have the form $\frac{\frac{1}{3}mg(x)}{5a}$ and $\frac{\frac{2}{3}mg(y)}{3a}$<br>M0 if extensions are assumed to be equal.                                                                                                                            |      |
| A1                          | Correct unsimplified equation with extensions that are unknown and sum to $2a$ .                                                                                                                                                                                                                                                  |      |
| A1*                         | Solve their equation correctly and use to obtain given answer from complete and correct working.                                                                                                                                                                                                                                  |      |
| <b>(b)</b>                  |                                                                                                                                                                                                                                                                                                                                   |      |
| M1                          | N2L at a general point. Dimensionally correct. Both tensions present. Condone sign errors but must be a difference in tensions. Accept acceleration as $a$ for the method mark only.<br>M0 if tensions are at a fixed point.<br>M0 if extensions are assumed to be equal.<br>M0 if resultant force is never used with $m\ddot{x}$ |      |
|                             |                                                                                                                                                                                                                                                                                                                                   |      |
| A1                          | Unsimplified equation with at most one error.<br>(if acceleration is not $\ddot{x}$ , this is one error)                                                                                                                                                                                                                          |      |
| A1                          | Correct unsimplified equation                                                                                                                                                                                                                                                                                                     |      |
| A1*                         | Reach given conclusion from complete and correct working. This must include all of: <ul style="list-style-type: none"> <li>at least one step between the initial N2L equation in <math>x</math> and the conclusion</li> <li>reaching the correct form <math>\ddot{x} = -\frac{g}{5a}x</math></li> <li>conclude: SHM</li> </ul>    |      |
| <b>(c)</b>                  |                                                                                                                                                                                                                                                                                                                                   |      |
| M1                          | Use of $\frac{2\pi}{\omega}$ with their $\omega$ or the correct $\omega$ substituted                                                                                                                                                                                                                                              |      |
| A1*                         | Given answer obtained using the correct $\omega$                                                                                                                                                                                                                                                                                  |      |

|                |                                                                                                                                                                                                                                    |  |
|----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <b>(d)</b>     |                                                                                                                                                                                                                                    |  |
| M1             | Correct method using $x = \pm a \sin \omega t$ to find a relevant time.<br>Must use $x = \frac{1}{3}a$ and amplitude $\frac{2}{3}a$ but no need to substitute for $\omega$<br>e.g. $x = \frac{2}{3}a \sin \omega t = \frac{1}{3}a$ |  |
| A1             | Correct value for a relevant time.                                                                                                                                                                                                 |  |
| A1             | Correct answer in exact form.                                                                                                                                                                                                      |  |
| <b>(d) ALT</b> |                                                                                                                                                                                                                                    |  |
| M1             | Correct method using $x = \pm a \cos \omega t$ to find a relevant time.<br>Must use $x = \frac{1}{3}a$ and amplitude $\frac{2}{3}a$ but no need to substitute for $\omega$<br>e.g. $x = \frac{2}{3}a \cos \omega t = \frac{1}{3}a$ |  |
| A1             | Correct value for a relevant time.                                                                                                                                                                                                 |  |
| A1             | Correct answer in exact form.                                                                                                                                                                                                      |  |