Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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Centre Number | | | | | |

Candidate Number | | | | |

# Pearson Edexcel International Advanced Level

## Thursday 6 June 2024

Morning (Time: 1 hour 30 minutes)

**Paper reference** **WMA14/01**

# Mathematics
## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over*





Pearson

1. **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

Find

$$\int_0^{\frac{\pi}{6}} x\cos 3x\,\mathrm{d}x$$

giving your answer in simplest form.

**(5)**

Question 1 continued

**(Total for Question 1 is 5 marks)**

**2.** With respect to a fixed origin, $O$, the point $A$ has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point $B$.

**(2)**

The point $C$ has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where $a$ is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of $a$.

**(4)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

3. The curve $C$ is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to $C$ at the point (2, 5), giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(7)**

Question 3 continued

**(Total for Question 3 is 7 marks)**

**4.**

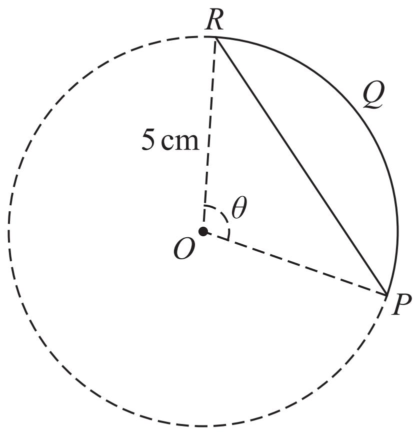


**Figure 1**

Figure 1 shows a sketch of a segment $PQRP$ of a circle with centre $O$ and radius 5 cm.

Given that

- angle $POR$ is $\theta$ radians
- $\theta$ is increasing, from 0 to $\pi$, at a constant rate of 0.1 radians per second
- the area of the segment $PQRP$ is $A\,\text{cm}^2$

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K\left(1 - \cos\theta\right)$$

where $K$ is a constant to be found.

**(2)**

(b) Find, in $\text{cm}^2\,\text{s}^{-1}$, the rate of increase of the area of the segment when $\theta = \dfrac{\pi}{3}$

**(4)**

**Question 4 continued**

**(Total for Question 4 is 6 marks)**

**5.**

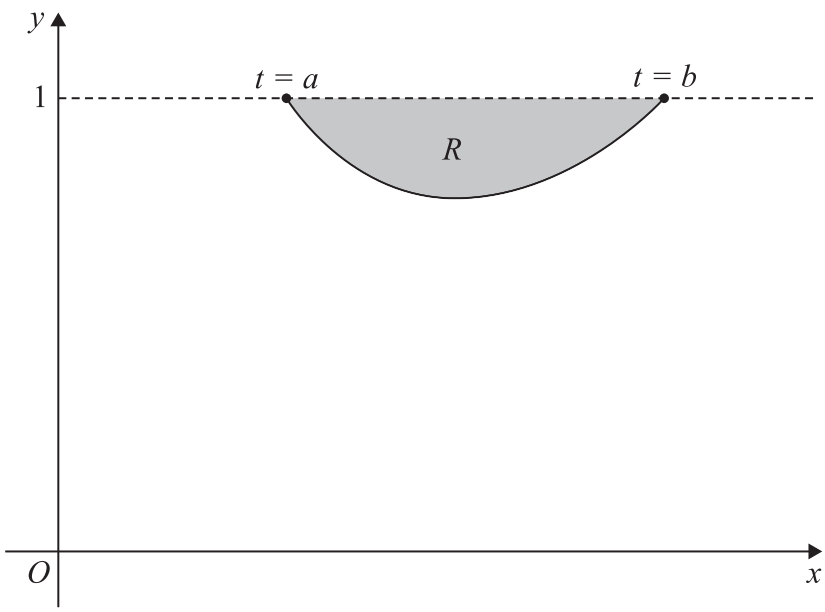


**Figure 2**

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where $a$ and $b$ are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of $a$ and the value of $b$

**(2)**

The region $R$, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region $R$ is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where $M$ and $k$ are constants to be found.

**(5)**

(c) (i) Write $\dfrac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of $R$, giving your answer in simplest form.

**(6)**

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P 7 4 3 1 5 R A 0 1 0 2 8

**Question 5 continued**

**Question 5 continued**

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Question 5 continued

**(Total for Question 5 is 13 marks)**

**6.** With respect to a fixed origin $O$, the line $l_1$ is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $A$ lies on $l_1$

Given that $|\overrightarrow{OA}| = 5\sqrt{10}$

(a) show that at $A$ the parameter $\lambda$ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

**(3)**

Hence

(b) (i) show that one possible position vector for $A$ is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for $A$.

**(3)**

The line $l_2$ is parallel to $l_1$ and passes through $O$.

Given that

- $\overrightarrow{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point $B$ lies on $l_2$ where $|\overrightarrow{OB}| = 4\sqrt{10}$

(c) find the area of triangle $OAB$, giving your answer to one decimal place.

**(4)**

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**Question 6 continued**

**Question 6 continued**

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Question 6 continued

(Total for Question 6 is 10 marks)

**7.** The current, $x$ amps, at time $t$ seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where $k$ is a constant.

Initially there is zero current in the circuit.

(a) Solve the differential equation to find an equation, in terms of $k$, for the current in the circuit at time $t$ seconds.
Give your answer in the form $x = f(t)$.

**(6)**

Given that in the long term the current in the circuit approaches 7 amps,

(b) find the value of $k$.

**(2)**

(c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 11 marks)**

**8.** $$f(x) = (8 - 3x)^{\frac{4}{3}} \qquad 0 < x < \frac{8}{3}$$

(a) Show that the binomial expansion of $f(x)$ in ascending powers of $x$ up to and including the term in $x^3$ is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \ldots$$

where $A$ and $B$ are constants to be found.

**(4)**

(b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \qquad 0 < x < \frac{8}{3}$$

where $A$ and $B$ are the constants found in part (a).

(*Solutions relying on calculator technology are not acceptable.*)

**(4)**

Question 8 continued

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 8 marks)**

**9.**

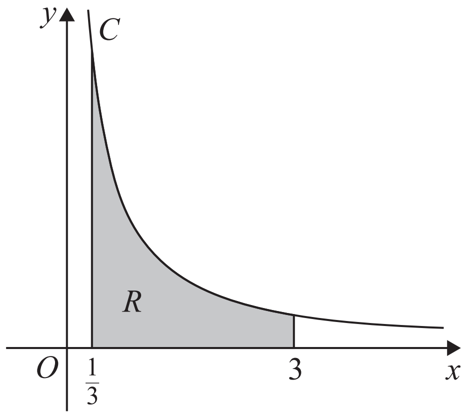


**Figure 3**

The curve $C$, shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\left(\arctan\sqrt{x}\right)}$$

The region $R$, shown shaded in Figure 3, is bounded by $C$, the line with equation $x = 3$, the $x$-axis and the line with equation $x = \frac{1}{3}$

The region $R$ is rotated through 360° about the $x$-axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume $V$ of the solid formed is given by

$$k\int_a^b \frac{1}{u^2}\,\mathrm{d}u$$

where $k$, $a$ and $b$ are constants to be found.

**(6)**

(b) Hence, using algebraic integration, find the value of $V$ in simplest form.

**(3)**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 9 marks)**

**TOTAL FOR PAPER IS 75 MARKS**