



# Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level  
In Pure Mathematics (WMA12) Paper 01

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## **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
  - ft – follow through
    - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC – special case
  - oe – or equivalent (and appropriate)
  - d... or dep – dependent
  - indep – independent
  - dp – decimal places
  - sf – significant figures
  - \* – The answer is printed on the paper or ag- answer given
  - $\square$  or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
  - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Pure Mathematics Marking

*(NB specific mark schemes may sometimes override these general principles)*

### Method mark for solving 3 term quadratic:

#### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$  leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$  leading to  $x = \dots$

#### 2. Formula

Attempt to use correct formula (with values for a, b and c)

#### 3. Completing the square

Solving  $x^2 + bx + c = 0$  :  $(x \pm \frac{b}{2})^2 \pm q \pm c$ ,  $q \neq 0$  leading to  $x = \dots$

### Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1 ( $x^n \rightarrow x^{n-1}$ )

#### 2. Integration

Power of at least one term increased by 1 ( $x^n \rightarrow x^{n+1}$ )

### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

### Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

### Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

### Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
<b>1. (a)</b>	$\left(1 - \frac{1}{6}x\right)^9 = 1 + 9\left(-\frac{1}{6}x\right)^1 + \frac{9 \times 8}{2}\left(-\frac{1}{6}x\right)^2 + \frac{9 \times 8 \times 7}{3!}\left(-\frac{1}{6}x\right)^3 + \dots$ $= 1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3$	M1 A1, A1 <b>(3)</b>
<b>(b)</b>	$(10x + 3)\left(1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3\right)$ <p>Coefficient of <math>x^3 = 10 \times 1 + 3 \times -\frac{7}{18} = \frac{53}{6}</math></p>	M1 A1 <b>(2)</b> <b>(5 marks)</b>

(a)

M1: For an attempt at the binomial expansion. Score for a correct attempt at any of term 2, 3 or 4.

Accept sight of  ${}^9C_1\left(\pm\frac{1}{6}x\right)^1$  or  ${}^9C_2\left(\pm\frac{1}{6}x\right)^2$  or  ${}^9C_3\left(\pm\frac{1}{6}x\right)^3$  condoning the omission of brackets.

Coefficients may appear from Pascal's triangle but should be correct for the term.

A1: For any two simplified terms of  $-\frac{3}{2}x + x^2 - \frac{7}{18}x^3$

A1: For  $1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3$  ignoring terms with greater powers. Allow as a list, allow  $+-\frac{3}{2}x$  etc but the fractions must be simplified. Accept  $1x^2$ . ISW following a correct expression (e.g. if multiply through by 2). If given as decimals they must be exact (recurring if appropriate).

(b)

M1: For attempting to find  $10b \pm 3c$  combined to a single term for their  $(10x + 3)\left(1 + ax + bx^2 + cx^3 + \dots\right)$

A1: cao for  $\frac{53}{6}$  but condone  $\frac{53}{6}x^3$  and isw. Accept embedded in an expansion with other terms as long as simplified to correct single  $x^3$  term. If given as a decimal must be exact recurring.

Question Number	Scheme	Marks
<b>2.(a)</b>	$a + 5d = 2$ and $\frac{10}{2}\{2a + 9d\} = -80$ Solves simultaneously $\Rightarrow d = 20, a = -98$	B1 B1 M1 A1 <b>(4)</b>
<b>(b)</b>	Attempts $\frac{n}{2}\{2 \times -98 + (n-1) \times 20\} \dots 8000$ $5n^2 - 54n - 4000 \dots 0 \Rightarrow n = (34.2)$ $n = 35$	M1 dM1 A1 <b>(3)</b> <b>(7 marks)</b>

(a)

B1: One of  $a + 5d = 2$  or  $\frac{10}{2}\{2a + 9d\} = -80$

B1: Both equations correct. May be un-simplified (isw after correct equations seen).

M1: Solves their two equations in  $a$  and  $d$  (one of which must be correct) and finds at least  $a$  or  $d$ . There may be slips, but score for any attempt that reaches a value provided at least one equation was correct and both attempted. Implied by correct answers following correct equations stated.

A1: For  $d = 20, a = -98$

**Alt**

B1: Correct formula in  $d$  only using 6<sup>th</sup> and 10<sup>th</sup> term  $\frac{10}{2}((2 - 5d) + (2 + 4d)) = -80$

B1:  $d = 20$

M1: Attempts to find  $a$  using  $a + 5d = 2$

A1: Correct  $a$

(b)

Note: allow all marks in (b) with any equality or inequality used throughout.

M1: Attempts to use  $\frac{n}{2}\{2 \times -98 + (n-1) \times 20\} \dots 8000$  with their  $a$  and  $d$ . Accept if set equal to 8001. NB If 800 or 80000 is used, allow the M (misread).

dM1: Sets up and **solves** a 3 term quadratic equation in  $n$ . Accept if just the positive root is given, but at least one positive root must be obtained.

Allow use of calculator /formulae (may need to check values if no method shown).

May be implied by a rounded or truncated positive integer value following a suitable equation having been set up.

FYI the correct equation is  $5n^2 - 54n - 4000 \dots 0$

A1:  $(n =) 35$  stated following a correct equation formed (allowing for incorrect inequality).

Alt by trial and improvement

M1: Attempts to use  $\frac{n}{2}\{2 \times -98 + (n-1) \times 20\}$  with their  $a$  and  $d$  and integer values of  $n$  either side of the sum being 8000

dM1: Two consecutive values for  $n$  giving results either side of 8000 attempted.

A1:  $n = 35$  stated following correct working with both  $n=34$  and  $n= 35$  having been seen.



Question Number	Scheme	Marks
<b>3 (i)</b>	$2 \log_2 (2-x) = 4 + \log_2 (x+10)$ One correct log law $2 \log_2 (2-x) \rightarrow \log_2 (2-x)^2$ or $4 \rightarrow \log_2 16$ Correct attempt to combine two terms Correct equation (not involving logs) E.g. $(2-x)^2 = 16(x+10)$ $x^2 - 20x - 156 = 0$ $(x+6)(x-26) = 0$ $x = -6$ only	B1 M1 A1 M1 A1 <b>(5)</b>
<b>(ii)</b>	12	B1 <b>(1)</b> <b>(6 marks)</b>

(i)

B1: One correct log law seen or used in relation to the equation e.g.  $2 \log_2 \dots \rightarrow \log_2 \dots^2$  or  $4 \rightarrow \log_2 16$  (which may appear as  $\log_2 2^4$  or  $\log_2 4^2$ ). May be seen anywhere and allow if on incorrect terms.

M1: **Correct** attempt to combine two of the **original** terms, condoning a slip with the 16. E.g.  $4 + \log_2 (x+10) = \log_2 16 + \log_2 (x+10) = \log_2 16(x+10)$   
Do not allow if e.g. they incorrectly divide by 2 first and attempt to combine  $\log_2 (2-x) - \log_2 (x+10)$

A1: Correct equation (not involving logs) from fully correct log work E.g.

$$(2-x)^2 = 16(x+10) \text{ or } \frac{(2-x)^2}{x+10} = 16$$

M1: Correct attempt to solve a 3TQ in  $x$  achieving a real root (may be by calculator). B1 must have been scored but not all log work need be correct.

A1:  $x = -6$  only following the award of all previous marks (all log work must have been correct). The  $x = 26$  solution must be rejected if seen.

SC Allow max B1M0A0M1A0 for e.g.  $\frac{\log_2(2-x)^2}{\log_2(x+10)} = 4 \rightarrow (2-x)^2 = 16(x+10)$  etc.

Allow the SC if no combination of logs is shown at all, but the correct quadratic is produced.

(ii)

B1: 12 clearly identified as the answer. Condone  $a = 12$  as long as 12 is clearly the answer being offered.

Question Number	Scheme	Marks
<b>4.(a)</b>	21	B1 (1)
<b>(b)</b>	Sets $f\left(\pm\frac{1}{2}\right) = 0 \rightarrow$ equation in $k$ Eg. $\left(\frac{1}{2}-2\right)\left(2\times\frac{1}{4}+5\times\frac{1}{2}+k\right)+21=0$ $\Rightarrow 3+k=14 \Rightarrow k=11 *$	M1 A1* (2)
<b>(c)</b>	(i) $f(x)=(x-2)(2x^2+5x+11)+21=2x^3+x^2+x-1$ $= (2x-1)(x^2+x+1)$ (ii) Attempts to find the number of roots of their $(x^2+x+1)$ <ul style="list-style-type: none"> <li>States <math>(x^2+x+1)</math> has no roots with reason</li> <li>Concludes <math>f(x)=0</math> has 1 root (at <math>x=\frac{1}{2}</math>)</li> </ul>	M1 dM1, A1 M1 A1 (5) (8 marks)

(a)

B1: States 21

(b) If division is attempted in (b), mark (b) and (c) as a whole.

M1: Sets  $f\left(\pm\frac{1}{2}\right) = 0 \rightarrow$  equation in  $k$ . The “= 0” may be implied. If division is used look for setting the remainder after dividing by  $(2x - 1)$  equal to zero to form an equation in  $k$ .

A1\*: Solves a correct equation in  $k$  showing at least one correct intermediate line (the “= 0” may be implied but must have been a correct equation preceding the answer).  
Condone missing brackets if clearly recovered.

May see  $k=11$  embedded so  $\left(-\frac{3}{2}\right)(3+11)+21=-21+21=0 \checkmark$  is acceptable.

(c) (i) Only allow work carried out in (b) to count in (c) if carried forward into (c).

M1: Attempts to multiply out  $(x-2)(2x^2+5x+11)+21$  to achieve a cubic. If an incorrect value for  $k$  is used, allow two M’s for equivalent work.

dM1: Attempts to divide by or factor out  $(2x-1)$  or  $\left(x-\frac{1}{2}\right)$  from their attempt at expanding  $(x-2)(2x^2+5x+11)+21$ .

Long division – look for the  $x^2$  + extra terms and first subtraction

$$\begin{array}{r}
 x^2 + \dots \\
 2x-1 \overline{) 2x^3 + x^2 + x - 1} \\
 \underline{2x^3 - x^2} \phantom{+ x - 1} \\
 2x^2 + x - 1 \\
 \underline{2x^2 + \dots} \\
 \dots
 \end{array}$$

Inspection/comparing coefficients – look for correct magnitudes for first and last term in the brackets.  $(2x-1)(x^2 \pm \dots \pm 1)$

A1:  $(2x-1)(x^2+x+1)$  from correct work. Must be brought together in one expression.

Accept  $2\left(x-\frac{1}{2}\right)(x^2+x+1)$  but not  $\left(x-\frac{1}{2}\right)(2x^2+2x+2)$  Condone if set equal to zero.

Award A0 if they subsequently incorrectly further factorise the quadratic term.

(c)(ii)

M1: Must have attempted to take the correct factor out of their cubic. Attempts to find the number of roots of their  $(x^2+x+1)$  **not of**  $2x^2+5x+11$  Allow for an attempt **any** of the following:

- an attempt at  $b^2-4ac$
- an attempt at the formula or completing the square
- correct (imaginary) roots from a calculator (must be seen)

Do not accept “no roots” (oe) seen with no evidence for why. Do not accept “Math Error” without an explanation of the error.

A1: Requires all of the following (or equivalents)

- correct factorisation  $f(x) = (2x-1)(x^2+x+1)$  or correct separate factors
- correct reason as to why  $(x^2+x+1) = 0$  hasn't any roots. E.g.  
 $b^2-4ac = 1-4(=-3) < 0$  hence no roots. There must be a (correct) calculation, reference to the sign of the discriminant and deduction/implication of no roots, or equivalent via other methods
- concludes  $f(x) = 0$  has one solution

Ignore erroneous extra statements after a correct reason has been given.

Some examples (following correct factorisation).

Scores M1A1:

$x^2+x+1 \rightarrow \Delta = 1-4 < 0$  no real solutions therefore  $x = \frac{1}{2}$  is only solution.

$b^2-4ac = -3 < 0$  so no solutions to  $x^2+x+1$  hence  $f(x)$  has only one real solution  $(2x-1)$ .  
*We ignore any reference to  $(2x-1)$  being the solution.*

Scores M1A0:

(ii) one real solution as  $x^2+x+1$  has  $b^2-4ac = -3$  so no real solution. *No reference to  $< 0$*

$b^2-4ac = 1^2-4(1)(1) = -3 \leq 0$  so there is only 1 real solution as  $x^2+x+1 \leq 0$  so the only real solution is  $(2x-1)$  which is  $x = \frac{1}{2}$  *Needs  $< 0$  to be stated*

$\frac{-1 \pm \sqrt{1^2-4(1)(1)}}{2}$  no solution,  $2x-1$ ,  $x = \frac{1}{2}$  is the only real solution. *Needs either complex roots stated (accepting  $\sqrt{-3}$  for  $3i$ ) or sign of discriminant considered.*

If you see an alternative approach that you think deserves credit, send to review.

Question Number	Scheme	Marks
<b>5 (a)</b>	$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$	M1
	$(x-y)^3 > x^3 - y^3 \Rightarrow -3x^2y + 3xy^2 > 0$	A1
	$\Rightarrow 3xy(y-x) > 0$	dM1
	As $x$ and $y$ are positive numbers $(y-x) > 0 \Rightarrow y > x$ *	A1*
		<b>(4)</b>
<b>(b)</b>	Chooses a suitable counter example. E.g. $x = 3, y = -1$	M1
	Shows that $(3 - (-1))^3 > (3)^3 - (-1)^3$ as $64 > 28$ (BUT $-1 < 3$ )	A1
		<b>(2)</b>
		<b>(6 marks)</b>

(a)

M1: Attempts  $(x-y)^3 = x^3 \pm Ax^2y \pm Bxy^2 \pm y^3$  but may be unsimplified. Condone at most one slip in an index as long as the middle terms have at least  $xy$  in them.

A1: Correct simplified inequality, cubed terms cancelled, other terms may not all be gathered.

E.g.  $-3x^2y + 3xy^2 > 0$  o.e. Terms need not be all on one side, e.g.  $3xy^2 > 3x^2y$

dM1: Takes out a common factor of  $kxy$  or divides by  $kxy$ . Must be clear division or stated as dividing, but dividing and rearranging to the given answer in one step from  $-3x^2y + 3xy^2 > 0$  to  $y > x$  is dM0 if method/intermediate step is not stated.

A1\*: Requires

- correct working (no incorrect lines) before achieving  $y > x$
- reasoning given in the correct/appropriate place,
  - e.g. as  $x$  and  $y$  are positive so  $3xy(y-x) > 0 \Rightarrow (y-x) > 0$

Alt

M1: Attempts  $(x-y)^3 = (x-y)(x^2 + axy + y^2)$  **and**  $x^3 - y^3 = (x-y)(x^2 + bxy + y^2)$  but may be unsimplified

A1: Both expressions correct  $(x-y)^3 = (x-y)(x^2 - 2xy + y^2)$  **and**

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

dM1: Takes to one side, takes out a common factor  $(x-y)$  (dividing by this term is M0) and cancels square terms.

A1\*: Requires

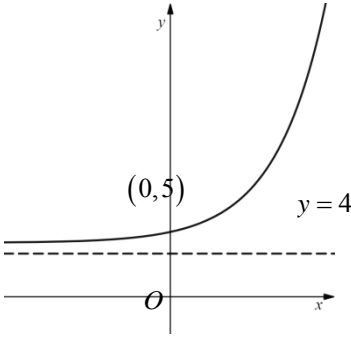
- correct working (no incorrect lines) before achieving  $y > x$
- reasoning given in the correct/appropriate place,
  - e.g. as  $x$  and  $y$  are positive so  $3xy(y-x) > 0 \Rightarrow (y-x) > 0$

(b)

M1: Chooses a suitable counter example. E.g.  $x = 3, y = -1$

Any positive  $x$  and negative  $y$  will work. Substitution is not needed for this mark. Note that if they choose  $x$  and  $y$  both positive it is M0.

A1: Shows that the result is not true for their values. As a minimum accept substitution into both sides of the inequality.

Question Number	Scheme	Marks
<b>6.(a)</b>	 <p>Shape <b>or</b> asymptote Intercept</p> <p>Fully correct</p> <p><math>h = 0.3</math></p>	<p>M1 B1 A1</p> <p>(3)</p>
<b>(b)</b>	$\text{Area} \approx \frac{0.3}{2} \{0 + 4.3137 + 2 \times (0.3246 + 0.8629 + 1.6643 + 2.7896)\}$ $= \text{awrt } 2.34$	<p>M1 A1</p> <p>(3)</p>
<b>(c) (i)</b>	$\int_2^{3.5} (2^x + 2x) dx = \int_2^{3.5} (2^x - 2x + 4x) dx = 2.34 + \left[ 2x^2 \right]_2^{3.5}$ $= 2.34 + 16.5 = 18.84$	<p>M1 A1ft</p>
<b>(ii)</b>	$\int_2^{3.5} (2^{x+1} - 4x) dx = \int_2^{3.5} 2(2^x - 2x) dx = 2 \times 2.34 = 4.68$	<p>B1 ft</p> <p>(3)</p>
		<b>(9 marks)</b>

(a)

- M1: For either the correct shape, an increasing curve in any position in quadrants 1 and 2 **or** for the correct asymptote (labelled in some way at height 4) but must have a curve approaching as an asymptote, not just the line drawn. Be tolerant on “pen slips” approaching the asymptote.
- B1: Correct intercept (touching or crossing). Accept 5 on axis, or stated as (0,5). Condone (5,0) on the graph if it is in the right place.
- A1: Fully correct. Shape in quadrants 1 and 2 only with the y intercept at 5 and asymptote of  $y = 4$  clearly drawn on the graph and labelled or stated separately. Be tolerant on “pen slips” approaching the asymptote. If there is conflicting information, what is on the graph takes precedence.

(b)

- B1: For  $h = 0.3$  This is implied by sight of  $\frac{0.3}{2}$  in front of the bracket.
- M1: Applies the trapezium rule with correct bracket condoning slips in copying (or even truncating) values. May use separate trapezia. Allow a missing final bracket, but otherwise bracketing must be correct or recovered by correct answer.
- A1: awrt 2.34 Note that the calculator answer for this integral is 2.30

(c) (i)

M1: For realising that  $\int_2^{3.5} (2^x + 2x) dx = \int_2^{3.5} (2^x - 2x + 4x) dx = 2.34 + \left[ kx^2 \right]_2^{3.5}$  The  $4x$  should be seen with an attempt to integrate (increase in power) made (though may be implied).  
 Could also use answer to (b) and add the area under the trapezium  $\frac{1.5}{2}(4 \times 2 + 4 \times 3.5)$  - may be implied by correct answer after seeing the integral correctly split.

Another alternative is to isolate the  $\int_2^{3.5} 2^x dx = \left[ x^2 \right]_2^{3.5} + "2.34"$  and use this to find a value for the integral required.

A1 ft: "2.34" + 16.5 = 18.84 but follow through on their 2.34 and allow awrt to 3s.f..

There must be a suitable method for this mark to be awarded.

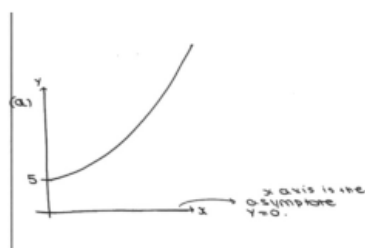
Answer via repeated trapezium rule scores M0A0

(c)(ii)

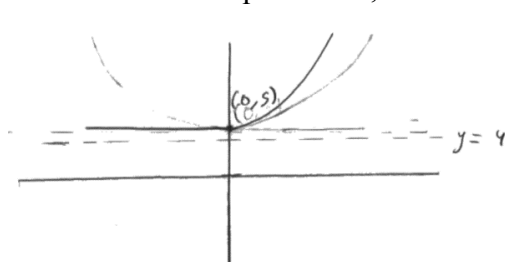
B1 ft: For  $2 \times$  their "2.34" = 4.68 ((awrt to 3s.f.) must be evaluated). Follow through on their answer to (b).

Note: Answer via repeated trapezium rule is permitted to score the B1 as long the answer is twice their (b).

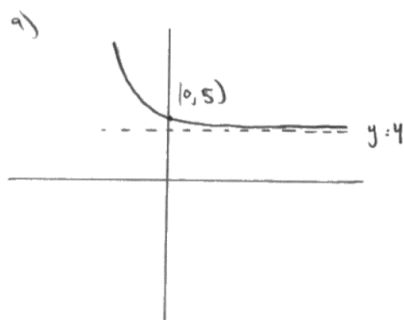
Some examples of graphs for (a):



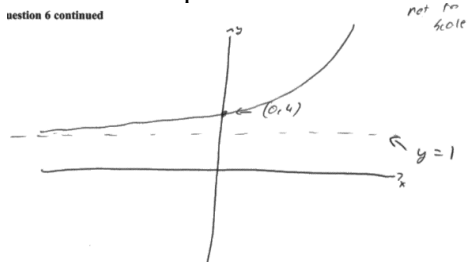
M0B1A0 Not in quadrant 2, but does have correct intercept.



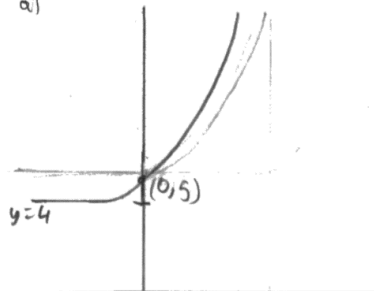
M1B1A0 Does not approach asymptote



M1B1A0 Correct asymptote and intercept



M1B0A0 Incorrect intercept and asymptote



M1B1A0 asymptote not drawn.

Question Number	Scheme	Marks
7 (a)	$x^2 + y^2 + 8x - 10y = 29$ Attempts $(x+4)^2 + (y-5)^2 \dots = \dots$ Correct centre $(-4, 5)$ Exact radius $\sqrt{70}$	M1 A1 A1 <b>(3)</b>
	(b) Attempts distance (or distance <sup>2</sup> ) from $(-4, 5)$ to $(5, -8)$ Full method. E.g. compares $\sqrt{9^2 + 13^2}$ to $(\sqrt{70} + \sqrt{52})$	M1 dM1
	States that <ul style="list-style-type: none"> <li>distance between centres <math>= \sqrt{250} = 15.81</math></li> <li>sum of radii <math>= \sqrt{70} + \sqrt{52} = 15.58</math></li> </ul> Concludes as sum of radii < distance between centres, result proven *	A1* <b>(3)</b> <b>(6 marks)</b>

(a)

M1: Attempts to complete the square for **both** variables. Look for  $(x \pm 4)^2, (y \pm 5)^2 \dots = \dots$   
or may be implied by  $(\pm 4, \pm 5)$  by inspection from the equation.

A1:  $(-4, 5)$

A1:  $\sqrt{70}$  which may be scored following  $(x \pm 4)^2 + (y \pm 5)^2 = 70$

(b)

M1: Attempts distance (or distance <sup>2</sup>) from  $(-4, 5)$  to  $(5, -8)$  (condone sign errors)

dM1: Full method. E.g. compares their  $\sqrt{9^2 + 13^2}$  to their  $(\sqrt{70} + \sqrt{52})$

A1\*: Requires correct calculation(s), correct reason and minimal conclusion. Must be evidence to support the inequality – e.g. decimal values – not just the inequality stated. May use the difference between sum of radii and distance between centres.

(b) Alt

M1: Attempts to solve  $x^2 + y^2 + 8x - 10y = 29$  and  $x^2 + y^2 - 10x + 16y = -37$  forming a linear equation (need not be simplified for this mark)

FYI: Linear equation is  $18x - 26y = 66$  o.e. if the calculations are correct

dM1: Attempts to solve their  $18x - 26y = 66$  simultaneously with either circle equation reaching a 3 term quadratic for either x or y.

A1\*: Requires

- correct equations
- correct "solution" stating no roots and concluding no points of intersection and minimal conclusion

E.g. Substitute  $y = \frac{9x-33}{13}$  into  $x^2 + y^2 + 8x - 10y = 29$  gives  $250x^2 - 412x + 478 = 0$

and  $b^2 - 4ac = -308256 < 0$  so no solutions

FYI the equation in y (if correct) is  $250y^2 + 984y + 1116 = 0$

Question Number	Scheme	Marks
<b>8 (i)</b>	States or uses $\tan x = \frac{\sin x}{\cos x} \Rightarrow 5 \sin x \times \frac{\sin x}{\cos x} + 13 = \cos x$ $5 \sin^2 x + 13 \cos x = \cos^2 x \Rightarrow 5(1 - \cos^2 x) + 13 \cos x = \cos^2 x$ $\Rightarrow 6 \cos^2 x - 13 \cos x - 5 = 0$ $\Rightarrow (3 \cos x + 1)(2 \cos x - 5) = 0 \Rightarrow \cos x = -\frac{1}{3}$ $\Rightarrow x = 1.91$	B1 M1 A1 M1 A1 <b>(5)</b>
<b>(ii) (a)</b>	$20 = 10 + 12 \sin(6k + 18)^\circ \Rightarrow \sin(6k + 18)^\circ = \frac{5}{6}$ $\Rightarrow (6k + 18) = 56.4, 123.6$ $\Rightarrow k = 6.41, 17.59$	M1 dM1 A1, A1 <b>(4)</b>
<b>(b)</b>	22°C	B1 <b>(1)</b>
<b>(c)</b>	"6.41"t + 18 = 90 $\Rightarrow t = 11.23$ Time of day = 11:14	M1, A1 <b>(2)</b> <b>(12 marks)</b>

(i)

B1: States or uses  $\tan x = \frac{\sin x}{\cos x}$       E.g.  $5 \sin x \times \frac{\sin x}{\cos x} + 13 = \cos x$

M1: Attempts to use  $\tan x = \frac{\sin x}{\cos x}$ ,  $\sin^2 x + \cos^2 x = 1$  and multiply by  $\cos x$  to form a quadratic equation in  $\cos x$  (allow if there are slips in coefficients but the trig terms must be correct)

A1: Correct simplified quadratic  $6 \cos^2 x - 13 \cos x - 5 (=0)$  The “=0” may be implied.

M1: Solves a 3TQ in  $\cos x$  leading to at least one value for  $\cos x$

A1: awrt  $x = 1.91$  (following  $\cos x = -\frac{1}{3}$ ) and no other values in the range.

Note: Answers only with no working score no marks. From  $6 \cos^2 x - 13 \cos x - 5 (=0)$  to 1.91 directly, score final M0A0.

(ii) (a)

M1: Attempts to use the given information and proceeds to  $\sin(6k + 18)^\circ = c$ , or may be implied by

$$A \sin(6k + 18)^\circ = B \rightarrow 6k + 18 = \arcsin \frac{B}{A} \text{ (evaluated) (which scores M1dM1)}$$

dM1: Takes arcsin leading to a value for  $6k + 18$ . Accept radian values seen for this mark. (0.985, 2.16).  
Allow awrt 2 s.f. answers for evidence.

A1: One value for  $k$ : 6.41 or 17.59 Must be in degrees.

SC Allow for 6.4 and 17.6 both given if no more accurate answers are stated.

A1: awrt  $k = 6.41, 17.59$  and no other values

Note answers only scores no marks. – the requirements of the M marks must be seen to be able to score them.

(ii)(b)

B1: cao 22°C Condone just 22.

(ii)(c)

M1: Sets their "6.41"t + 18 = 90  $\Rightarrow t = \dots$  May be implied by 11.23.

$$\text{Note "6.41"t + 18} = \frac{\pi}{2} \text{ is M0.}$$

A1: cao Time of day = 11:14 o.e. (e.g. 11h 14m is acceptable).



Question Number	Scheme	Marks
<b>9.(a)</b>	$y = 2x^{\frac{3}{2}}(4-x) = 8x^{\frac{3}{2}} - 2x^{\frac{5}{2}}$ $\frac{dy}{dx} = 12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}$ $\text{Stationary point } 12x^{\frac{1}{2}} - 5x^{\frac{3}{2}} = 0 \Rightarrow x = \frac{12}{5}$	M1, A1 dM1, A1 <b>(4)</b>
<b>(b)</b>	$\int 8x^{\frac{3}{2}} - 2x^{\frac{5}{2}} dx = \frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}}$ $\frac{16}{5}k^{\frac{5}{2}} - \frac{4}{7}k^{\frac{7}{2}} = 0 \Rightarrow k = \frac{16 \times 7}{4 \times 5} = \frac{28}{5}$	M1, A1 dM1, A1 <b>(4)</b> <b>(8 marks)</b>

(a)

M1: Complete attempt to differentiate.

Writes as a sum of two terms and reduces the power of at least one term by 1

Alternatively, by the product rule look for  $Ax^{\frac{1}{2}}(4-x) - Bx^{\frac{3}{2}}$

A1:  $\left(\frac{dy}{dx} = \right) 12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}$  or  $3x^{\frac{1}{2}}(4-x) - 2x^{\frac{3}{2}}$ . May be left unsimplified

dM1: Sets  $\frac{dy}{dx} = 0$  and attempts to solve to find a non-zero value for  $x$ . Do not be concerned about the method of solving for this mark.

A1cso:  $x = \frac{12}{5}$  from correct work. Ignore reference to  $x = 0$ . Watch for incorrect methods of

solving, such as  $\left(12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}\right)^2 = 0 \Rightarrow 144x - 25x^3 = 0 \Rightarrow x = \frac{12}{5}$ , which score A0.

(b)

M1: Complete attempt to integrate. Writes as a sum of two terms and increases the power of at least one term by 1. For attempts at integrating by parts they must proceed through the second integration to score the mark.

A1:  $\frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}}$  which may be unsimplified

dM1: Substitutes  $x = k$  in their  $\frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}} = 0$  and solves. Alternatively, applies limits 0 and 4 to find the area  $R_1 \left( = \frac{1024}{35} \right)$  and sets equal to the negative of the expression from limits 4 to  $k$  (or reverses the limits and sets equal) and proceeds to reach a value for  $k$ .

A1:  $k = \frac{28}{5}$  (ignore references to  $k = 0$ ) Accept as decimal 5.6.

Alt for the dM:

$$\text{Area } R_1 = \left[ \frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_0^4 = \frac{1024}{35} \Rightarrow \text{Area } R_2 = \left[ \frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_4^k = -\frac{1024}{35} \Rightarrow k = \dots$$

Note if  $\text{Area } R_1 = \text{Area } R_2 = \left[ \frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_0^4 = \frac{1024}{35} = \left[ \frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_4^k \Rightarrow k = \dots$  is used then

this scores dM0. However, if a student realises the areas cancel and effectively achieves the correct equation after stating areas equal then allow recovery for dM1A1.

Question Number	Scheme	Marks
<b>10 (a)</b>	$2000 \times 1.03^6 = \text{awrt } 2390$	M1, A1 (2)
<b>(b)</b>	$3690 = ab^4, 3470 = ab^7 \Rightarrow b^3 = \frac{3470}{3690} = (0.94\dots)$ Hence $b = \sqrt[3]{\frac{3470}{3690}} = 0.9797\dots \quad a = \frac{3690}{0.9797\dots^4}$ $N = 4000 \times 0.98^t$	M1 dM1 A1 (3)
<b>(c)</b>	Sets $2000 \times 1.03^T = 4000 \times 0.98^T \Rightarrow \left(\frac{1.03}{0.98}\right)^T = \frac{4000}{2000}$ $\Rightarrow T \log\left(\frac{1.03}{0.98}\right) = \log 2 \Rightarrow T = \dots$ $\Rightarrow T = 13.9$	M1 dM1 A1 (3) (8 marks)

(a)

M1: Attempts  $2000 \times 1.03^5$  or  $2000 \times 1.03^6$ . May use successive iterations (finding each year) but end up at one these in effect.

A1: Awrt 2390

(b)

M1: Makes progress in using the model and attempting to find one of the variables  $a$  or  $b^3$  (or  $b$ ). Allow if  $r$  is used instead of  $b$  for the M's.

Allow from the indexing error  $3690 = ab^3, 3470 = ab^6 \Rightarrow b^3 = \frac{3470}{3690}$

dM1: Full method to find values for  $a$  and  $b$ .

Condone this being scored from  $3690 = ab^3, 3470 = ab^6$

A1:  $N = 4000 \times 0.98^t$  Accept awrt 4000 to 2sf., awrt 0.98. Must be seen as an equation not just values of  $a$  and  $b$  but allow if the equation is seen (c) (including the  $N =$ ).  
Condone  $n$  or  $T$  used the variables.

(c)

M1: Makes progress towards solving the problem by forming a single term with single power  $T$ . Follow through on their equations provided that they are of the correct form.

Alternatively takes logs of both sides  $2000 \times 1.03^T = 4000 \times 0.98^T$  and uses addition law and power law of logs, e.g.  $\ln 2000 + T \ln 1.03 = \ln 4000 + T \ln 0.98$

Award from an attempt on  $2000 \times 1.03^{T-1} = 4000 \times 0.98^T \Rightarrow \left(\frac{1.03}{0.98}\right)^T = \frac{4000 \times 1.03}{2000}$  or similar index error.

dM1: Full and complete attempt to find  $T$ . Dependent upon previous method but may be implied for the correct value for their  $a = b^T$

A1: cao  $T = 13.9$

